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MODIFIED FORMULA FOR ESTIMATING THE BOWEN RATIO – DESCRIPTION OF THE DETERMINATION PROCEDURE

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ABSTRACT

Aim of the study
One way of estimating evapotranspiration is to use the heat balance structure of active surface.

Material and methods
The paper shows how to derive a formula estimating the Bowen ratio by using measurement data of heat balance structure of various crops, and in the next step, apply that ratio to estimate evapotranspiration. A two-stage method is described, in which the first step is to determine empirical index W, and the second step is to calculate the Bowen ratio. This allows the researcher to see the results in a graphic format, while searching for the best outcome, and to implement simple tools offered in spreadsheets.

Results and conclusions
The obtained formulas made it possible to estimate evapotranspiration of agricultural crops with high accuracy in relation to the measured value.

Keywords: active surface heat balance components, Bowen ratio, modelling

INTRODUCTION

In their publication, Rosa et al. (2020) analyse the global water shortage in agriculture, referring to the increase in demand for food, with the crucial influence of water and nutrients (Mueller et al., 2012). They point out good access to nitrogen fertilizers (Erisman et al., 2008), with simultaneous, increasingly limited access to water (D'Odorico et al., 2018), which will force farmers to use irrigation as a result (Rosa et al., 2018). The consequence will be further burdening of fresh water resources, which in turn raises concerns in terms of its limited availability (Rockström et al., 2012; Elliott et al., 2014).

Therefore, it is very important to monitor evapotranspiration, which depends on the condition of the vegetation cover and on the course of the weather, so that irrigation is used only when necessary, and in the most effective way. This harmonizes very well with the requirements of economy, because irrigation is an expensive agricultural procedure.

One of the available methods for evapotranspiration estimation is the use of heat balance structure of active surface, which can be used both during evaporation measurements (Leśny et al., 2021; Xiong et al., 2022), and during its estimation (Leśny, 1998, 2011; Souza et al., 2023). This paper shows how to employ

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measurement data of the heat balance structure of various crops to derive a formula estimating the Bowen ratio, and how to use the aforementioned ratio to estimate evapotranspiration in the next step.

MATERIALS AND METHODS

Measurements of heat balance structure used in the present work were conducted in several places in Europe over the course of 5 years. The geographical location and meteorological conditions of the measurement sites in Poland, Germany, France, and Spain are given in Table 1.

In each of the places listed in Table 1, on agricultural fields, measurements of heat balance structure were made using the Bowen method, which was described in detail in the work of Leśny et al. (2021). A total of 116 full measurement days were used for the analyses, and calculations were made using the average daily values of heat balance components given in $\frac{w}{m^2}$. Based on the data obtained in this way, the Bowen ratio (Bowen, 1926) was determined, Subsequently, the parameterization of the Bowen ratio – as mentioned in the introduction – can be used to model the structure of heat balance of active surface.

In Olejnik's works (1988): two-stage Bowen ratio estimation was proposed. In the first step, an empirical indicator was adopted:

$$W_o = \frac{100 \cdot \left(D \cdot \sqrt{V}\right)^{arctg\left(\frac{\text{À}}{2} \cdot f\right)}}{t \cdot \left(0.4 + u\right)} \tag{1}$$

where:

- W_o – empirical index,
- D – water vapor pressure deficiency (hPa),
- V – wind speed at a height of 2 m above ground level (m/s),
- f – degree of plant development assumed in the range of ⟨0.1⟩, determined as the ratio of the current value of leaf area index to the maximum value for a given type of crop,
- t – air temperature (ºC),
- u – relative insolation i.e. the ratio of the length of time of arrival of direct radiation to the length of the day (dimensionless value),

In the second step, the following equation for the Bowen ratio was proposed:

$$\beta_o\left(W_o\right) = \frac{13.23}{W_o + 3.9} - 0.02 \tag{2}$$

It was found that the observed correlations between standard meteorological data and the Bowen ratio are so substantial and statistically significant that the given equations can be successfully used to estimate the components of heat balance in much larger areas. However, it was noted that it would be advisable to conduct further research on these correlations (Olejnik, 1988).

In the present paper, the aforementioned results were treated as an inspiration for further research. One of the first steps was to calculate the value of the W_o indicator for the analysed data set. Statistically,

Table 1. Geographical coordinates of six measurement sites with the length of the growing season as the period between the month when the temperature exceeds 5ºC and the month when the temperature drops below 5ºC (source: elaborated by the Authors)

Place of measurements	Country	Symbol	Latitude	Longitude	Altitude	The growing season
Biebrza	Poland	B	53º40'N	22 º30'E	120 m	IV–X
Laufen	Germany	L	47º50'N	13 º00'E	500 m	IV–X
Turew	Poland	T	52º30'N	17º00'E	80 m	IV–X
Müncheberg	Germany	M	52º35'N	14º10'E	80 m	IV–X
Cessieres	France	C	49º30'N	3º30'E	60 m	III–XI
Saragossa	Espanola	Z	41º30'N	0º50'W	250 m	I–XII

an analogous equation describing the dependence of the Bowen ratio on the value of W_o was re-derived. To do this, a simple tool included in spreadsheets was applied, namely, a procedure that allows determining linear regression coefficients. Equation 2 can be represented in the general form:

$$\beta(W) = A \cdot \frac{1}{W + B} + C \tag{3}$$

Using the linear regression tool to find the values of parameters A and C, assuming that the set of dependent variables are the measured values of the Bowen ratio $\{\beta\}$ and the independent variables are the set of indices $\{W\}$ determined on the basis of measured meteorological parameters, additionally transformed to the form $\frac{1}{W + B}$, the B parameter is now selected additionally. The latter selection is a procedure for searching the solution for which the linear regression is the best fit, for instance, for the automatic finding of R^2. In this case, one of the procedures contained in spreadsheets to optimize the parameter value for a given criterion can be used successfully. The procedure described herein seems almost primitive, but it is very simple and, with relatively small data sets, it allows the researcher to quickly obtain results, and to easily illustrate them directly in the spreadsheet used. Of course, more sophisticated data analysis packages can be applied, but they require much more knowledge, and they are often included in software packages that require a paid subscription. It also seems that sometimes researchers who use advanced tools treat them very instrumentally, which may lead to drawing too superficial conclusions.

RESULTS

The formula determined according to the method described in the methodology section took the following form:

$$\beta_w(W_o) = \frac{11.145}{W + 0.17} - 0.148, \quad R^2 = 0.64 \tag{4}$$

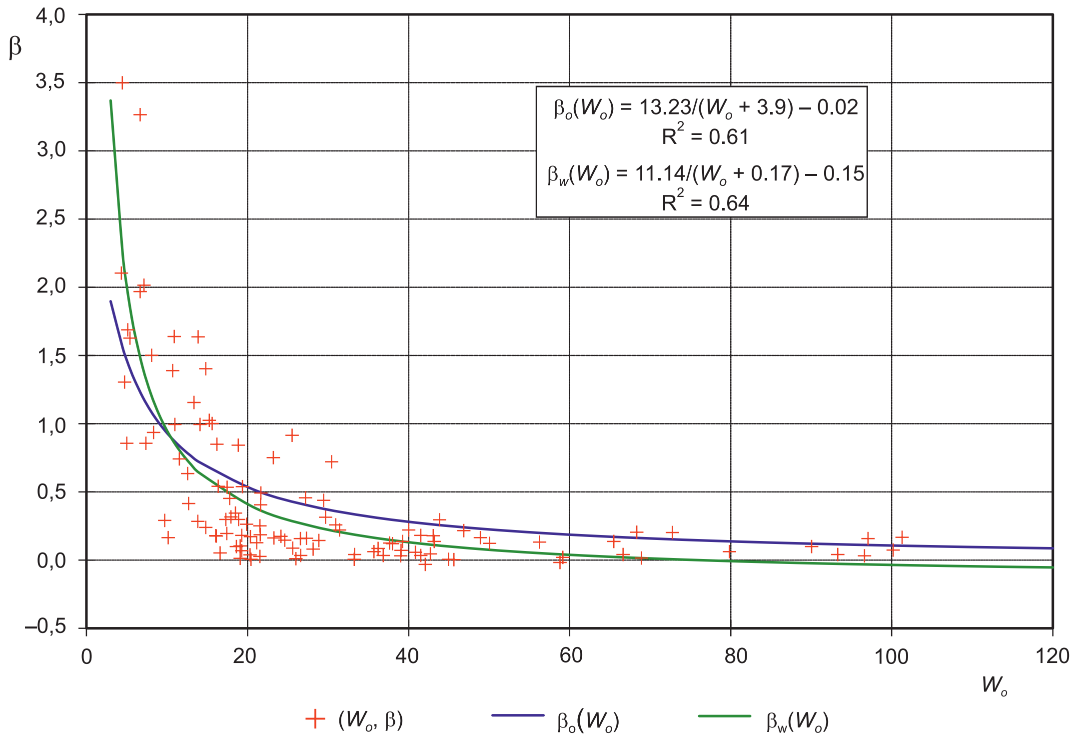

Fig. 1. Graphic illustration of a set of points with coordinates (W_o, β), and graphs of the functions $\beta_o(W_o)$ and $\beta_w(W_o)$ (source: elaborated by the Authors)

Figure 1 graphically illustrates the obtained set of points. The abscissa is the W_o value calculated with formula 1, the ordinate is the Bowen ratio obtained from the measurements of the components of heat balance, and the solid lines show the dependencies of the Bowen ratio on the W_o index, determined by adopting equations 2 and 4.

Although the obtained approximation is relatively good, further research was deemed appropriate – chiefly because a much larger set of data from the measurements of the components of heat balance of active surface was available. The data was obtained using new hardware (Leśny et al., 2021), therefore, the results are likely to have a smaller measurement error. In addition, the research was carried out in many places with different climatic conditions and on multiple types of field crops.

It was decided that in order to determine the directions of further research, the impact of individual meteorological parameters and plant development phases on the Bowen ratio should be analysed. For this purpose, the coefficient R^2 was calculated for each pair of sets: parameter – Bowen ratio, and drawings illustrating these relationships were made.

The results are presented in Figure 2, revealing that the only quantity that individually shows a large impact on the Bowen ratio is the degree of plant development. The coefficient of determination R^2 for this quantity takes the value of 0.39, thus the correlation r = 0.63. For the remaining parameters, the correlation is insignificant. However, after substituting these variables into formula 1 and determining W_o, it can be observed that there is a much higher correlation between the related parameters and the value of the Bowen ratio.

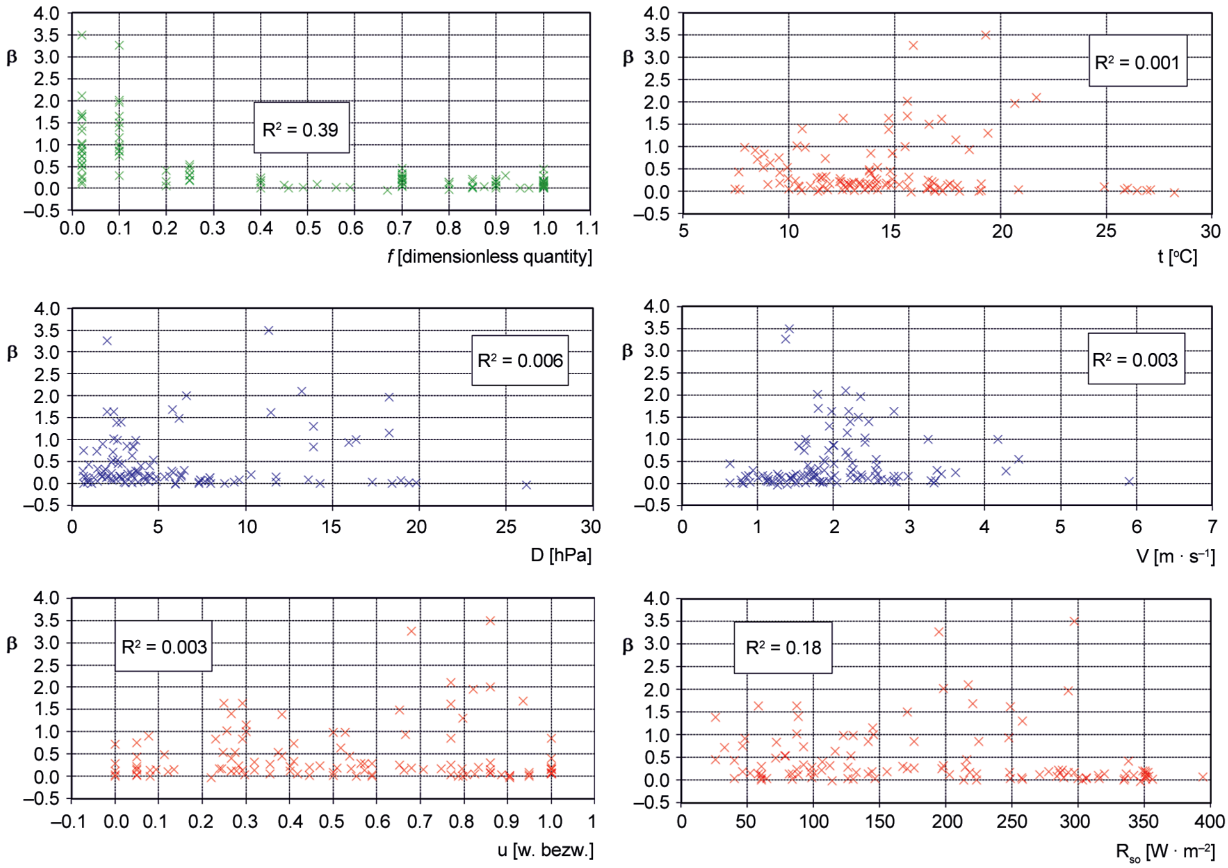

Fig. 2. The correlation of the Bowen ratio – from top left: with the development phase of plants (f, β), with air temperature (t, β), with humidity deficiency (D, β), with wind velocity (V β), with relative insolation (u, β), and with total radiation (R_{SO}, β) (source: elaborated by the Authors)

Therefore, it was concluded that further modifications of the agro-meteorological index would be appropriate. A larger data set will probably make it more universal, and it will facilitate achieving better results of approximations of the Bowen ratio using functions describing its dependence on the W index.

As a result of multiple transformations of the formula for the agro-meteorological index and attempts to take into account other meteorological elements, the final formula for calculating the *W* index was obtained, producing a better dependence of the Bowen ratio (β) on this index than if it were calculated according to the original formula. From the parameters, only irradiation was added to the horizontal surface at the boundary of the atmosphere R_0, which is a value depending on the season and latitude.

Finally, the equation defining the comprehensive agro-meteorological index took the following form:

$$W_u = \frac{10 \cdot V}{t} \cdot \left(\frac{D \cdot \sqrt{R_o}}{0.4 + u} \right)^{arctg(0.33 \cdot \text{À} \cdot f)} \quad (5)$$

the function representing the values of the *W* indicator then took the form:

$$\beta_u(W_u) = \frac{1.671}{W - 0.33} + 0.042, \quad R^2 = 0.76 \quad (6)$$

Equation (5) cannot be assessed using diagnostic measures, because it is only a consolidation of a number of measurement data, whereas the method for determining the parameters of equation (6) is described in the last paragraph of the methodology section. This procedure also allowed us to determine the coefficient of determination given above ($R^2 = 0.76$). The mean square error determined on the basis of this equation, and for all 116 measurement data, is 0.095. For the sake of comparison, if formulas (1) and (2) were used, respectively, this error would be 0.178.

The resulting set of points is illustrated in Figure 3, where the solid line is the graph of the function $\beta_u(W_u)$. This function was chosen to approximate the value of the Bowen ratio because, from the very beginning of the analysis of the dataset, the shape of the

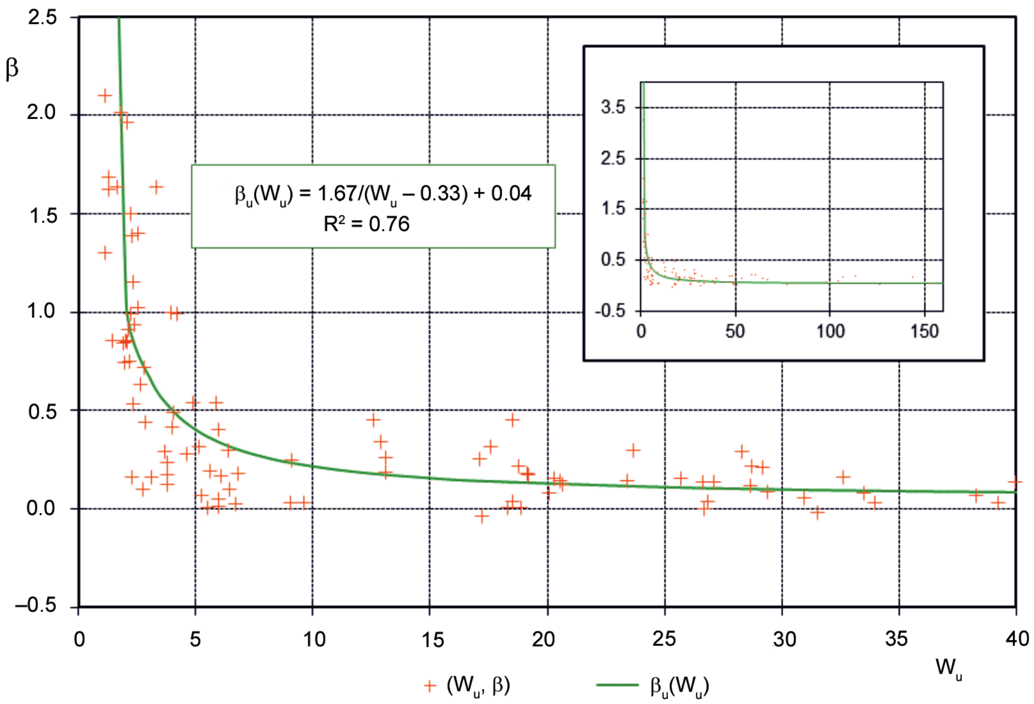

Figure 3. The correlation between the Bowen ratio and the agro-meteorological index (W_u, β), and the graph of the function $\beta_u(W_u)$ (source: elaborated by the Authors)

point cloud with coordinates (W_u, β) closely resembled a hyperbola plot. Other types of functions were also tried, but the values of β were best approximated by the hyperbola.

One of the arguments of the formula defining the agrometeorological index is insolation, however, we need to take into account the fact that total radiation is often measured at meteorological stations. Automation of measurements at meteorological station is significant here, as automatic sets for measuring standard meteorological data usually include a pyranometer. In addition, knowledge of total radiation carries more information about the energy supplied to the environment by means of short-wave radiation than knowledge of insolation. Since during the research on the structure of the heat balance, total radiation was also measured on most measurement days, in the formula describing the comprehensive agro-meteorological index, it was decided that relative insolation should be replaced with total radiation. In the place of insolation (u), total radiation (R_{so}) was inserted, then, with some simplifications, the procedure of searching for such coefficients and powers of parameters was repeated so that the statistically derived function β(W) would approximate the values of the measured Bowen ratios as closely as possible, as described in the last paragraph of the methodology section, and as mentioned when describing equations (5) and (6). The following structure of the formula describing the meteorological index was obtained, marked as W_R:

$$W_R = \frac{10 \cdot V}{t} \cdot \left(\frac{D \cdot \sqrt{R_o}}{0.75 + \frac{R_{so}}{400}} \right)^{arctg(0.33 \cdot \pi \cdot f)} \tag{7}$$

The corresponding function β(W) took the form:

$$\beta_R(W_R) = \frac{1.651}{W - 0.34} + 0.031 \quad R^2 = 0.75 \tag{8}$$

In this case, the results are not shown in the figure, because they would be practically identical to those from Figure 3. The obtained coefficient of determination ($R^2 = 0.75$) is slightly lower than for formula (6), whereas the mean square error for the measurement data, as can be expected, is slightly higher and amounts to 0.097. It is worth noting that the determined formulas can be used interchangeably, without detriment to the obtained results, depending on whether the available set of meteorological data includes relative insolation or total radiation.

DISCUSSION

In order to check the accuracy of the proposed empirical functions, the following procedure was carried out:

- using the proposed functions, Bowen ratios were determined for all measurement days,
- on that basis, values of the energy stream used for evapotranspiration (LE) were determined,
- the obtained values were converted into water equivalents of energy used for evaporation and their sum was calculated.

The procedure was carried out both for functions based on insolation and total radiation. The obtained pairing sums are presented in the table below:

$$\sum_{i=1}^{116} ETR_{ui} = 329.4 \ \text{[mm]},$$

$$\sum_{i=1}^{116} ETR_{Ri} = 328.3 \ \text{[mm]},$$

$$\sum_{i=1}^{116} ETR_{i} = 333.3 \ \text{[mm]},$$

where:

$ETR_{ui,Ri,i}$ – evapotranspiration calculated with the use of insolation data, total radiation, or measured on the i-th measurement day, respectively.

The method of analysing the collected measurement data presented in this chapter is based on statistical procedures, and the derived equations describe the relationships in an empirical way. Having analysed the form of the agro-meteorological index, we observe that the degree of plant development has a very large impact on its value. It should be added, however, that no parameter in the equations describing and determines the type of vegetation cover.

CONCLUSIONS

The methodology presented in the paper shows a simple way of determining the formulas for parameter estimation based on several measurement quantities. In this particular case, formulas for estimating the Bowen ratio were modified, and can be used to estimate evapotranspiration based on meteorological data and the stage of plant development.

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MODYFIKACJA FORMUŁY DO SZACOWANIA STOSUNKU BOWENA – OPIS PROCEDURY WYZNACZANIA

ABSTRAKT

Cel pracy

Jednym ze sposobów szacowania ewapotranspiracji jest wykorzystanie struktury bilansu cieplnego powierzchni czynnej.

Materiał i metody

W pracy pokazano, jak wykorzystując dane pomiarowe struktury bilansu cieplnego różnych upraw, wyprowadzić wzór szacujący stosunek Bowena, który w kolejnym kroku może posłużyć właśnie do oszacowania ewapotranspiracji. Opisano dwuetapową metodę, w której najpierw wyznacza się empiryczny wskaźnik W, a dopiero w drugim kroku stosunek Bowena. Pozwala to badaczowi graficznie zobaczyć uzyskiwane rezultaty w trakcie poszukiwania dobrego wyniku i wykorzystać proste narzędzia oferowane w arkuszach kalkulacyjnych.

Wyniki i wnioski

Uzyskane formuły z dużą dokładnością, w stosunku do zmierzonego, pozwalały oszacować ewapotranspirację upraw rolniczych.

Słowa kluczowe: bilans cieplny powierzchni czynnej, stosunek Bowena, modelowanie