ISSN 1644-0765
DOI: http://dx.doi.org/10.15576/ASP.FC/2015.14.1.57
www.acta.media.pl

SCIENTIARUM POLONORUM ACTA *Acta Sci. Pol. Formatio Circumiectus 14 (1) 2015, 57–73*

ANALYSIS OF THE HISTORICAL TRUSS IN VILLAGE BELÁ DULICE

Jozef Gocál, Peter Krušinský, Eva Capková, Miloš Kekeliak
University of Žilina

Abstract. The truss of the Roman Catholic Church of the Holiest Christ's Body was managed to date to the year 1409. It represents one of the few well-preserved medieval structures in this region. The sharp roof above the nave has a typical rafter collar-beam construction with longitudinal stiffening truss. The geometrical analysis of the main roof truss as well as the central longitudinal truss is based on logical dependencies and a description of a process in the truss design, pointing to evaluative relations resulting especially from the Pythagorean Geometry. Consequently, a spatial numerical model of the roof structure was developed in order to perform a static analysis of the roof structure in accordance with present standards. Due to the fact that during the diagnostic survey there were noted some missing structural elements in the roof construction (angle braces), in further analysis, an attention was paid to the importance of the selected structural elements and their role in the construction of the truss itself.

Key words: historical truss, geometric analysis, irrational proportion, static analysis, numerical model

INTRODUCTION

There have been published geometric analyses of many historical buildings up to now, which dealt almost exclusively with floor plans, cross sections and details, without consideration of the truss construction (e.g. in the monograph by Alojz Struhár [1977]), while the geometric analysis of the truss constructions and the way of their designing was marginalized, because of narrow specialization and the lack of knowledge. The research, based on the geometric analysis, results from the area investigation of the historical trusses in the Slovak Republic, which has been gradually realized from 2008 and provides a database of historical truss constructions for their geometric analysis and following statistic evaluation.

Corresponding authors: ing. Jozef Gocál Ph. D., ing. arch. Peter Krušinský Ph. D., ing. Miloš Kekeliak, Faculty of Civil Engineering, RNDr. Eva Capková Ph. D., Faculty of Humanities, University of Žilina, Univerzitná 8215/1, 010 26 Žilina, Slovakia; e-mail: jozef.gocal@fstav.uniza.sk, krusop@fstav.uniza.sk, milos.kekeliak@fstav.uniza.sk, eva.capkova@fhv.uniza.sk.

The first church chosen for the geometric analysis is the Church of the Holiest Christ's Body in Belá Dulice, built in 1409, that is typologically clean with a minimal manipulation during its existence. The main idea is to compare the origin geometric design concept with the present requirements for reliability of the truss structures.

BASIC DESCRIPTION OF THE CHURCH AND ITS TRUSS

The Roman Catholic church of the Holiest Christ's Body was built in the historical part of the village as a solitaire in the centre of a bordered complex and was oriented to the direction West-East. The church was probably established in the first half of the 14th century. There are two independent constructions found in the church, above the nave and above the sanctuary. In the analysis, we focused on the truss above the nave.

The sharp roof above the nave has a typical rafter collar-beam construction with longitudinal stiffening truss, dated to 1409. It contains four main trusses (every third truss is the main one) and six secondary trusses. In the main roof truss, the collar-beams cross the central king posts, which are lapped together with the rafters in the vertex. Symmetrical braces stabilize the king post and connect it to the tie beam. The rafters are mortised to the ends of the tie beam. Tall angle braces are used for the transversal bracing. All the joints are secured by wooden dowels. The central longitudinal truss consists of a sill beam lying on the tie beams, four king posts, strutted in the 3/5 of their height by horizontal braces and stabilised in the bottom end by symmetrical braces, and diagonal bracings placed in the upper part of the king posts.

GEOMETRIC ANALYSIS OF THE TRUSS CONSTRUCTION

Basis of the geometrical analysis

On designing Gothic buildings there were used such principles that reflected the state of knowledge of the builders. During the Gothic period, it was common to use geometry and arithmetic as a tool, as it is mentioned in the manuscript of Abbot Suger [2006] about the construction of the Gothic church in Saint-Denis (it is considered to be the first Gothic building). Geometry was one of the main pillars of their knowledge and the Pythagorean theory of proportions was a part of the seven liberal arts. Already Vitruvius [2001] based his work on the Pythagorean tradition, when in addition to the proportions of human body he recommended the use of so called musical ratios (especially 5 : 3 and 3 : 2) and the irrational proportion, numerically expressed by $\sqrt{2} : 1$, which specifies the relationship between the diagonal and the side of a square. Palladio [1797] recommended seven shapes of rooms to builders, in which musical ratios were used as well as the irrational proportion: a circular or square room, a rectangular room with a ratio of length and width: $\sqrt{2} : 1$, 4 : 3, 3 : 2, 5 : 3 and 2 : 1.

In the geometric analysis of the Gothic truss of the Roman-Catholic Church in the Slovak village Bela Dulice, which was dated by dendrochronology according to the period of used trees (European larch) to the year 1409 [Krušinský et al. 2008], we have identified some of the above ratios. As the basic ratio there was recognised the irrational proportion √2 : 1, which is closely related to the floor plan dimensions. Individual proportional relationships, excepting the irrational proportion, were determined by using the simple construction of so called eight-pointed star, the detail description of which can be found in [Brunés 1967].

Geometric analysis of the main truss

In the geometric analysis of main full truss above the nave features the square ABCD of the same size as in the floor plan (Fig. 1). The square ABCD creates one quarter of the square in the floor plan. Its side has a length equal to the half of the the width of the truss above the nave, the height of the truss (point V) was obtained by the same construction as in the case of the floor plan of the presbytery, i.e. using the circumference 1k with the centre in the point C and passing through the point of intersection of the diagonals of the square ABCD. The relationship between the width and the height of the truss above the nave can be numerically expressed as a ratio: 2 : (1 + √2/2). The slope of the roof, as well as the rafter AV, is thus defined.

The collar tie is located at a height that is equal to the length of the side of the basic square ABCD. The king post BV is divided by the collar tie in point C in the ratio √2 : 1. The location of the collar tie on rafters (points E, F) can be obtained by the construction of the circumference 2k (the centre of the circumference is point C and the radius is apparent from the figure 2) that cuts the king post in point G. Point G together with the circumference ^{2}k will be the starting point for the analysis of the longitudinal frame truss.

The endings of raking braces on rafters (points H, I) are located in the height equal to the half of the length of the side of the basic square ABCD, and in one quarter of the length

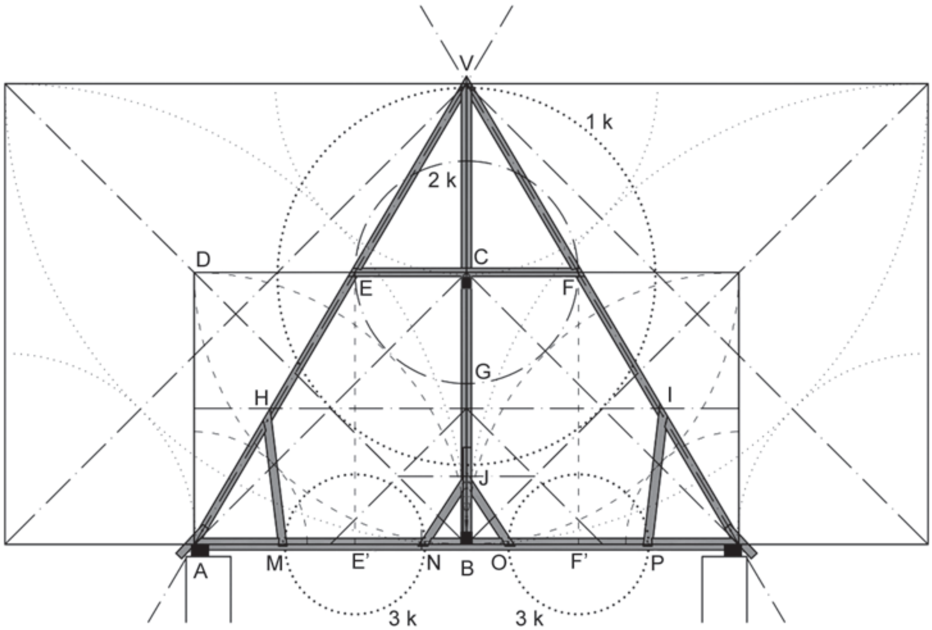

Fig. 1. Geometric analysis of the main truss

of the side of the basic square ABCD is the ending of the raking braces J on the king post. The points E’, F’ on the tie beam have the same distance from the point B as points E, F from the point C, i.e. they are perpendicular projections of the points E, F on the tie beam. The circumference 3k with the centre E’ and the radius equal to one quarter of the length of the basic square intersects the tie beam in the endings of raking braces M, N (symmetrically the circumference with the centre in F’ intersects the tie beam in the points O, P).

Geometric analysis of the central longitudinal truss

The analysis of a longitudinal frame truss above the nave is connected with the geometric elements in the transversal direction. The circumference *k* (C, r = |CE|) with the same centre and radius as in the main roof truss cuts the king post in the point G, which is the bottom end of the diagonal wind braces so called Ondrej’s crosses (Fig. 2). The intersection of wind braces and at the same time their axis of symmetry divides the entire height of the truss in the ratio of 3 : 2, on the basis of the axis of symmetry are constructed upper endings of the wind braces G´. Endings of raking braces on the king post are located in one-fifth of the total height of the truss and their endings on the tie beam are located on extensions of the wind braces.

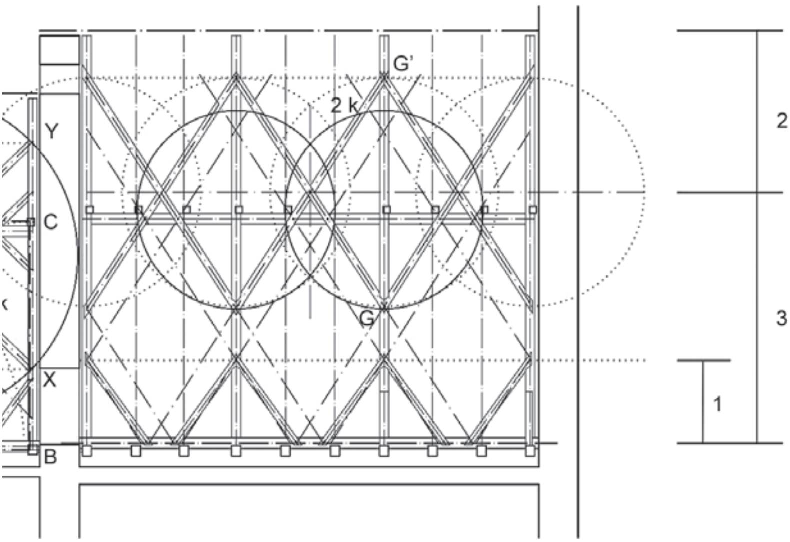

Fig. 2. Geometric analysis of central longitudinal frame truss

STATIC ANALYSIS OF THE TRUSS

Geometry of the numerical model

The numerical model of the roof structure was developed in the finite element analysis (FEA) software Scia Engineer [2010]. The roof structure is modelled as a three dimensional structure with beam elements. All member connections are modelled as hinge joints.

Considering the stresses in the roof members and structural parameters being analysed, the isotropic material model with mechanical properties of C24 according to STN EN 338 [2004] is adopted, which corresponds to the applied type of wood (larch). The geometry of numerical model is related to the roof structure's geometrical analysis presented hereinbefore. A perspective visualization of the numerical model is presented in Fig. 3. The basic geometric parameters of the numerical model are shown in Fig. 4 and Fig. 5. The cross sections of members are designated in the form b × h [mm], where "b" is the width and "h" is the height of the cross section in millimetres.

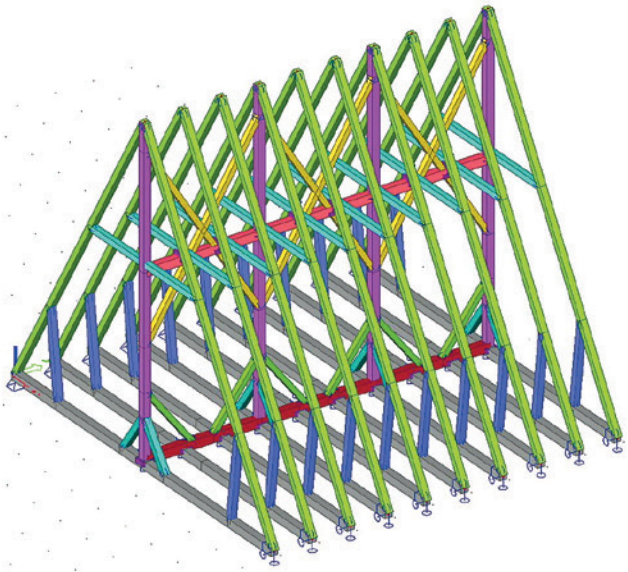

Fig. 3. Numerical model of the roof structure – 3D visualization

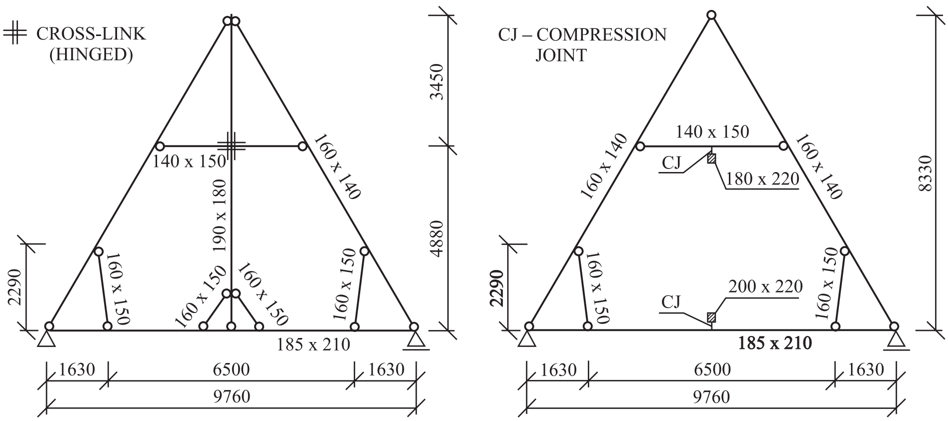

Fig. 4. Geometric parameters of the numerical model – the main cross truss and the secondary cross truss

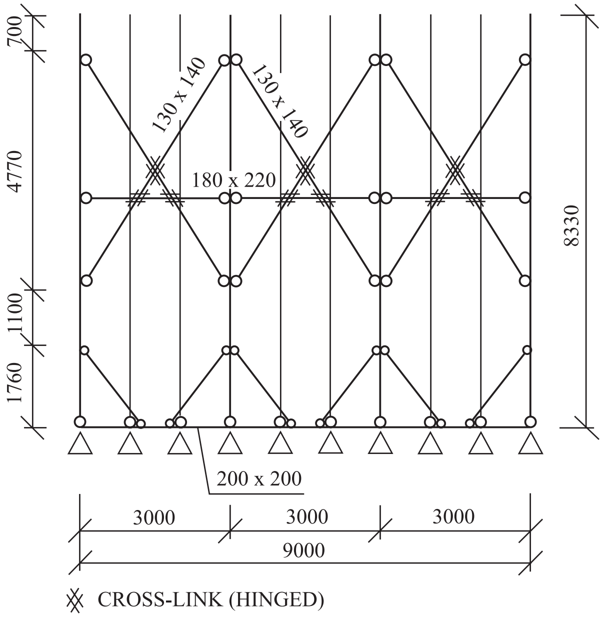

Fig. 5. Geometric parameters of the numerical model – the longitudinal truss

The loads

The roof superstructure was loaded according to standards STN EN 1990 [2009], STN EN 1991-1-1 [2007a] and STN EN 1991-1-4 [2007b] by permanent load (the self--weight and the weight of roofing) and variable load (wind actions). The self-weight load was generated by the FEA software. The other loads are applied on the rafters as uniformly distributed line loads. With regard to the locality of the structure, four wind load cases are considered for the wind region II and terrain category III. Reference height for the wind action is 15 m. With regard to the roof pitch angle, the snow load was not applied on the roof. The load configurations in particular load cases are visualised in Fig. 6. With regard to the considered load cases, ten combinations of load cases for ultimate limit states (denoted as NC1–NC10) and five combinations of load cases for serviceability limit states (denoted as NC11–NC15) were generated according to the standard STN EN 1990 [2009].

Modelling of joints

All the member connections are modelled as hinge joints with axial rigid connection and with capability of initial slip of 1 mm in the axial direction of member. The collar beams and tie beams of the secondary trusses are connected to the upper and bottom chords of the longitudinal truss by joints, which are able to transfer only compression forces (designated as CJ – compression joints – in Fig. 3). The initial slip is used to consider theoretical influence of gaps, cracks and geometry imperfections, occurring in historical carpentry joints. The imperfections are related with rheology

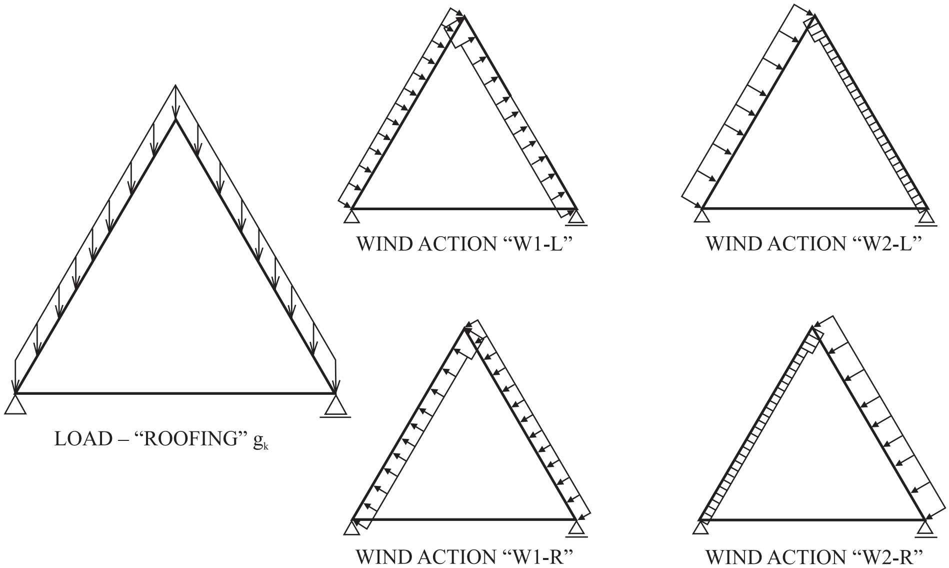

Fig. 6. Load configuration in load cases

of the wood material and its time dependent volume changes due to creep, including deformation of wood in joint areas. Modelling of joints with the slip capability can provide more realistic view on behaviour of existing historical timber structure, where the deformation at the end of lifetime should be assessed. The roof structure was built in 1409, what means that lifetime of 50 years defined by current European standards was 11-times exceeded.

Parametric study

During visual inspection of the roof structure, there were notified, except for other defects, several missing angle braces of the roof trusses at the supports and at the central columns. For this reason, a parametric study by means of the numerical model was performed in order to analyse the roof structural behaviour with and without particular timber truss members and to evaluate the differences in displacements and stress values in the main, secondary and longitudinal trusses of the roof structure. Three numerical models have been developed in all to simulate three kinds of roof trusses. The first numerical model denoted as “M0” represents the model of ideal roof structure with all timber members. The second numerical model denoted as “M1” is considered without angle braces near the supports of truss. Finally, the third numerical model denoted as “M2” is characterised by missing angle braces near the central column. The schemes of all numerical models are visualised in Fig. 7. The members of the longitudinal truss have been retained without any modifications.

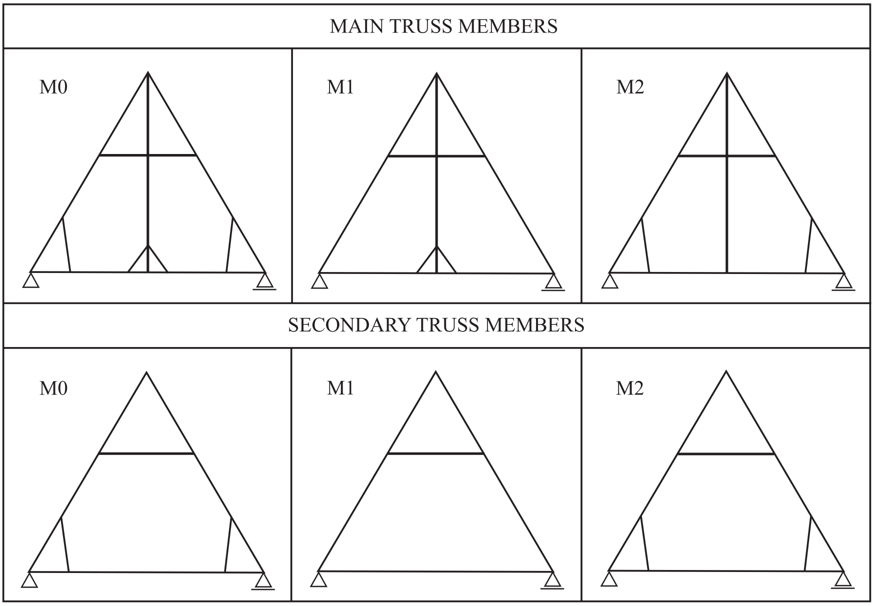

Fig. 7. Numerical models description, M0, M1, M2

THE NUMERICAL ANALYSIS RESULTS

The results of numerical analysis are presented by the values of maximum tensile and compression stresses in the roof members and displacements of trusses, calculated for the decisive load combinations. The convention for truss displacements direction used in Tab. 2, Tab. 4 and Tab. 6 is presented in Fig. 8. A negative value of the collar beam displacement means displacement of the central part of the collar beam vertically down. The maximum stresses in the main truss members and the displacements of members are presented in Tab. 1 and Tab. 2, respectively. The maximum stresses in the secondary truss members and the displacements of members are presented in Tab. 3 and Tab. 4, respectively. The maximum stresses in the longitudinal truss members and the displacement of bottom chord are presented in Tab. 5 and Tab. 6, respectively.

The maximum stresses in the roof members were caused by the wind actions. The stresses calculated for the load combination NC4 represent the maximum values almost for all roof members in all the numerical models. For some members, the maximums stresses have been calculated for the load combinations NC3, NC7 and NC8, containing the wind actions. The stresses calculated for the particular load combinations are presented in Tab.1, Tab. 3 and Tab. 5. It can be seen that the stresses obtained by numerical analysis of the complete roof structure (model M0) do not exceed the design values of the bending and the compression strength equal to 16.60 MPa and 14.53 MPa (timber strength class C24; $k_{mod} = 0.9$; $\gamma_M = 1.3$), defined in standard STN EN 1995-1-1 [2008]. In the case of the numerical model M1, where the absence of support angle braces is simulated, some of the stresses exceed these design strengths.

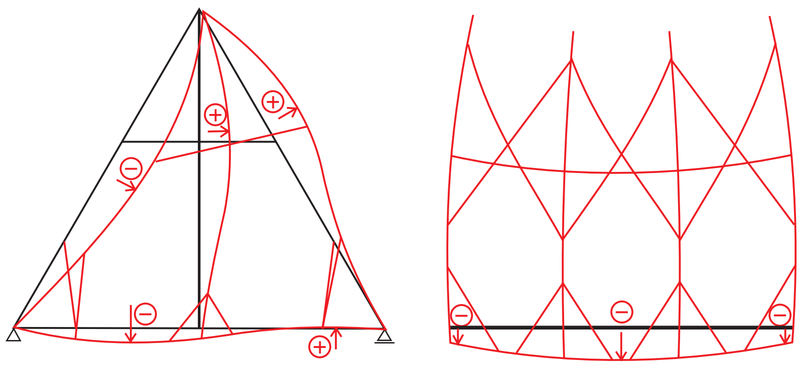

Fig. 8. Truss displacement – convention of displacement directions

Table 1. Normal stresses in the main truss members

Load Combination	Numerical model	Maximum normal stresses in members, MPa											
		Rafters		Collar beam		Central column		Support angle braces		Tie beam		Central angle braces	
		min.	max.	min.	max.	min.	max.	min.	max.	min.	max.	min.	max.
NC1	M0	–2.38	2.00	–7.77	7.42	–0.03	0.16	–0.01	0.02	–1.78	1.98	–0.03	0,03
	M1	–2.19	1.80	–7.85	7.52	–0.13	0.18			–1.77	1.98	–0.03	0,03
	M2	–2.34	1.96	–7.55	7.21	–0.13	0.18	–0.01	0.02	–1.72	1.92		
Ratio M1/M0		0.92	0.90	1.01	1.01	4.33	1.13	–	–	0.99	1.00	1.00	1.00
Ratio M2/M0		0.98	0.98	0.97	0.94	4.33	1.13	1.00	1.00	0.97	0.97	–	–
NC3	M0	–4.91	3.76	–6.63	6.30	–4.38	4.39	–0.19	0.23	–3.30	3.88	–0.24	0.23
	M1	–7.10	6.78	–8.34	8.02	–8.52	8.56			–4.66	5.06	–0.33	0.37
	M2	–5.25	4.54	–6.50	6.17	–4.40	4.42	–0.22	0.26	–4.55	5.11		
Ratio M1/M0		1.45	1.80	1.26	1.27	1.95	1.95	–	–	1.41	1.30	1.38	1.61
Ratio M2/M0		1.07	1.21	0.98	0.98	1.01	1.01	1.16	1.13	1.38	1.32	–	–
NC4	M0	–6.28	5.62	–10.13	9.40	–4.41	4.48	–0.25	0.18	–4.36	4.95	–0.19	0,27
	M1	–10.18	9.63	–8.36	7.63	–8.58	8.62			–4.74	5.26	–0.34	0,37
	M2	–8.36	7.45	–8.15	7.43	–4.39	4.43	–0.26	0.22	–5.62	6.17		
Ratio M1/M0		1.62	1.71	0.83	0.81	1.95	1.92	–	–	1.09	1.06	1.79	1.37
Ratio M2/M0		1.33	1.33	0.81	0.79	0.99	0.99	1.04	1.22	1.29	1.25	–	–

Table 1.cd.

Load Combination	Numerical model	Maximum normal stresses in members, MPa											
		Rafters		Collar beam		Central column		Support angle braces		Tie beam		Central angle braces	
		min.	max.	min.	max.	min.	max.	min.	max.	min.	max.	min.	max.
NC7	M0	–4.21	3.42	–5.01	4.77	–4.46	4.47	–0.18	0.23	–2.93	3.47	–0.24	0,23
	M1	–6.59	6.46	–6.14	5.91	–8.47	8.50			–4.27	4.73	–0.33	0,36
	M2	–4.87	4.31	–4.90	4.66	–4.38	4.39	–0.21	0.25	–4.34	4.84		
Ratio M1/M0		1.57	1.89	1.23	1.24	1.90	1.90	–	–	1.46	1.36	1.38	1.57
Ratio M2/M0		1.16	1.26	0.98	0.98	0.98	0.98	1.17	1.09	1.48	1.40	–	–
NC8	M0	–5.73	4.94	–8.18	7.56	–4.38	4.44	–0.25	0.17	–4.29	4.83	–0.19	0.27
	M1	–9.68	9.31	–6.16	5.52	–8.54	8.57			–4.35	4.82	–0.34	0.36
	M2	–7.74	6.92	–6.12	5.49	–4.42	4.45	–0.25	0.21	–5.30	5.81		
Ratio M1/M0		1.69	1.89	0.75	0.73	1.95	1.93	–	–	1.01	1.00	1.79	1.33
Ratio M2/M0		1.35	1.40	0.75	0.73	1.01	1.00	1.00	1.24	1.24	1.20	–	–

It is important to note that the buckling of members was not taken into account in the evaluation of the roof superstructure. Considering the fact that the share of compression stresses induced by the compression forces is minimal (caused by permanent loads only), it is possible to neglect this effect.

Table 2. Displacements of the main truss members

Load Combination	Numerical model	Displacement of truss, mm							
		Rafters		Collar beam		Central column		Tie beam	
		min.	max.	min.	max.	min.	max.	min.	max.
NC11	M0	–5.9	–	–11.4	–	–	–	–12.4	–
	M1	–5.4	–	–11.1	–	–	–	–12.1	–
	M2	–5.9	–	–11.0	–	–	–	–12.0	–
Ratio M1/M0		0.92	–	0.97	–	–	–	0.98	–
Ratio M2/M0		1.00	–	0.97	–	–	–	0.97	–
NC12	M0	–18.0	15.1	–9.8	–	–	13.7	–10.5	–
	M1	–39.2	33.1	–11.1	–	–	26.7	–13.1	–
	M2	–24.0	21.1	–9.6	–	–	19.5	–11.9	0,1
Ratio M1/M0		2.18	2.19	1.13	–	–	1.95	1.25	–
Ratio M2/M0		1.33	1.40	0.98	–	–	1.42	1.13	–
NC13	M0	–20.0	14.9	–13.6	–	–	13.8	–14.3	–
	M1	–45.4	29.5	–11.5	–	–	27.0	–13.5	–
	M2	–25.7	21.2	–11.5	–	–	19.6	–17.3	–
Ratio M1/M0		2.27	1.98	0.85	–	–	1.96	0.94	–
Ratio M2/M0		1.29	1.42	0.85	–	–	1.42	1.21	–

Table 3. Normal stresses in the secondary truss members

Load Combination	Numerical model	Maximum normal stresses in members [MPa]							
		Rafters		Collar beam		Support angle braces		Tie beam	
		min.	max.	min.	max.	min.	max.	min.	max.
NC1	M0	–2.21	1.99	–0.59	0.34	–0.01	0.03	–1.88	2.04
	M1	–2.04	1.76	–0.58	0.35	–	–	–1.86	2.01
	M2	–2.25	1.95	–0.58	0.34	–0.01	0.02	–1.80	1,96
Ratio M1/M0		0.92	0.88	0.98	1.03	–	–	0.99	0.99
Ratio M2/M0		1.02	0.98	0.98	1.00	1.00	0.67	0.96	0.96
NC3	M0	–8.46	7.83	–0.59	0.34	–0.29	0.36	–6.10	6.59
	M1	–17.42	16.61	–0.57	0.36			–1.63	2.09
	M2	–8.41	7.79	–0.59	0.34	–0.30	0.35	–6.12	6.62
Ratio M1/M0		2.06	2.12	0.97	1.06	–	–	0.27	0.32
Ratio M2/M0		0.99	1.00	1.00	1.00	1.03	0.97	1.00	1.01
NC4	M0	–11.54	10.78	–0.77	0.15	–0.34	0.31	–7.13	7.62
	M1	–21.30	20.36	–0.78	0.15			–1.69	2.15
	M2	–11.54	10.78	–0.77	0.15	–0.34	0.31	–7.13	7.62
Ratio M1/M0		1.85	1.89	1.01	1.00	–	–	0.24	0.28
Ratio M2/M0		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
NC7	M0	–7.71	7.17	–0.43	0.25	–0.30	0.34	–6.02	6.48
	M1	–16.89	16.15	–0.42	0.27			–1.20	1.62
	M2	–7.68	7.14	–0.43	0.25	–0.30	0.34	–6.03	6.49
Ratio M1/M0		2.19	2.25	0.98	1.08	–	–	0.20	0.25
Ratio M2/M0		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
NC8	M0	–10.88	10.19	–0.62	0.07	–0.34	0.30	–7.01	7.46
	M1	–20.75	19.87	–0.63	0.06			–1.25	1.68
	M2	–10.88	10.19	–0.62	0.07	–0.34	0.30	–7.01	7.46
Ratio M1/M0		1.91	1.95	1.02	0.86	–	–	0.18	0.23
Ratio M2/M0		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

The values of displacements are presented in Tab. 2, Tab. 4 and Tab. 6. In all the numerical models, the maximum displacements of almost all the roof members have been calculated for the load combination NC13, which contains the wind actions. For some members, the maximum displacements occurred for load combination NC11, which consists of permanent loads only.

Table 4. Displacements of the secondary truss members

Load Combination	Numerical model	Displacement of truss [mm]					
		Rafters		Collar beam		Tie beam	
		min.	max.	min.	max.	min.	max.
NC11	M0	–5.9	–	–2.6	–	–12.6	–
	M1	–5.4	–	–2.6	–	–12.3	–
	M2	–5.9	–	–2.6	–	–12.2	–
Ratio M1/M0		0.92	–	1.00	–	0.98	–
Ratio M2/M0		1.00	–	1.00	–	0.97	–
NC12	M0	–36.7	35.2	–1.4	–	–13.4	1.5
	M1	–137.2	131.2	–2.6	–	–11.7	–
	M2	–36.7	35.2	–1.4	–	–13.4	1.5
Ratio M1/M0		3.74	–	1.86	–	0.87	–
Ratio M2/M0		1.00	–	1.00	–	1.00	–
NC13	M0	–36.9	35.3	–1.5	–	–17.6	–
	M1	–140.4	130.1	–2.6	–	–12.1	–
	M2	–36.9	35.3	–1.5	–	–17.4	–
Ratio M1/M0		3.81	–	1.73	–	0.69	–
Ratio M2/M0		1.00	–	1.00	–	0.99	–

Table 5. Normal stresses in the Longitudinal truss members

Load Combination	Numerical model	Maximum normal stresses in members [MPa]							
		Upper chord		Bottom chord		X-Bracing – upper		X-Bracing – bottom	
		min.	max.	min.	max.	min.	max.	min.	max.
NC1	M0	–0.20	0.19	–0.33	0.33	–0.44	0.43	–0.07	0.07
	M1	–0.20	0.20	–0.37	0.37	–0.42	0.41	–0.07	0.07
	M2	–0.20	0.20	–0.35	0.36	–0.42	0.41	–0.07	0.07
Ratio M1/M0		1.00	1.05	1.12	1.12	0.96	0.95	1.00	1.00
Ratio M2/M0		1.00	1.05	1.06	1.09	0.96	0.95	1.00	1.00
NC3	M0	–1.18	1.18	–0.16	0.17	–3.01	3.01	–0.07	0.07
	M1	–2.25	2.24	–0.37	0.38	–5.77	5.77	–0.07	0.07
	M2	–1.40	1.39	–0.25	0.20	–3.49	3.49	–0.07	0.07
Ratio M1/M0		1.91	1.90	2.31	2.24	1.92	1.92	1.00	1.00
Ratio M2/M0		1.19	1.18	1.56	1.18	1.16	1.16	1.00	1.00
NC4	M0	–1.19	1.18	–0.39	0.39	–3.02	3.01	–0.07	0.07
	M1	–2.26	2.25	–0.36	0.36	–5.82	5.82	–0.07	0.07
	M2	–1.39	1.38	–0.47	0.48	–3.40	3.39	–0.07	0.07
Ratio M1/M0		1.90	1.91	0.92	0.92	1.93	1.93	1.00	1.00
Ratio M2/M0		1.17	1.17	1.21	1.23	1.13	1.13	1.00	1.00

Table 5. cd.

Load Combination	Numerical model	Maximum normal stresses in members [MPa]							
		Upper chord		Bottom chord		X-Bracing – upper		X-Bracing – bottom	
		min.	max.	min.	max.	min.	max.	min.	max.
NC7	M0	–1.16	1.15	–0.16	0.17	–3.03	3.03	–0.05	0.05
	M1	–2.19	2.19	–0.28	0.28	–5.70	5.70	–0.05	0.05
	M2	–1.36	1.35	–0.14	0.14	–3.43	3.43	–0.05	0.05
Ratio M1/M0		1.89	1.90	1.75	1.65	1.88	1.88	1.00	1.00
Ratio M2/M0		1.17	1.17	0.88	0.82	1.13	1.13	1.00	1.00
NC8	M0	–1.14	1.13	–0.29	0.30	–2.92	2.92	–0.05	0.05
	M1	–2.21	2.20	–0.26	0.27	–5.75	5.75	–0.05	0.05
	M2	–1.36	1.36	–0.37	0.37	–3.43	3.43	–0.05	0.05
Ratio M1/M0		1.94	1.95	0.90	0.90	1.97	1.97	1.00	1.00
Ratio M2/M0		1.19	1.20	1.28	1.23	1.18	1.18	1.00	1.00

Table 6. Displacements of the Longitudinal truss members

Load Combination	Numerical model	Displacement of Longitudinal truss	
		Begin and end position of bottom chord	Midspan of bottom chord
NC11	M0	–10.8	–12,6
	M1	–10.6	–12,4
	M2	–10.3	–12,2
Ratio M1/M0		0.98	0.98
Ratio M2/M0		0.95	0.97
NC12	M0	–9.3	–9.6
	M1	–10.0	–11.7
	M2	–9.3	–9.5
Ratio M1/M0		1.08	1.22
Ratio M2/M0		1.00	0.99
NC13	M0	–12.7	–14.3
	M1	–10.4	–12.1
	M2	–10.6	–12.9
Ratio M1/M0		0.82	0.85
Ratio M2/M0		0.84	0.90

Numerical model M1 vs. M0 – Absence of support angle braces

Main and secondary truss

The stresses in rafters of the main and the secondary truss have increased approximately 2-times, what shows a supporting effect of the angle braces. The increments of stresses occurred for the load combinations containing the wind actions, what indicates the main contribution of angle braces to force distribution when loaded by wind. The stresses of collar beams in main truss and secondary truss have increased by 27% and 8%, respectively. It can be seen that the stresses in collar beam of the main truss have been increased mainly for load combination containing the wind actions, as a consequence of a gap, that occurred between the upper chord of longitudinal truss and the collar beams of the secondary trusses. This gap developed as a consequence of the connection joint modelled with only compression capability and prevented to distribute the tensional and lateral shear forces into collar beams.

The stresses in central column have increased to double value, what is significant value. This increase indicates the effect of missing support angle braces. In case of load combination NC1 with permanent loads only the stresses increased 4-times, but the stresses calculated in this combination were low in terms of structural resistance of member.

The stresses in the main truss tie beam have increased 1.5-times. The tie beams have been loaded by the higher forces through the central column. It can be seen that the stresses in the tie beams of the secondary truss have decreased to value of approximately 2 MPa, what shows the tie beam as inactive truss member in terms of distribution of roofing and wind loads.

The central angle braces connecting the central column and the tie beam reached low values of stresses at all load cases. Although the stresses were low, the ratio of computed stresses (M1/M0) equal to value of 1.8 shows that the absence of support angle braces resulted in an increase of the forces in the central angle braces.

It can be observed, that the distribution of forces and lateral stiffness provided by the support angle braces in the original structure plays a crucial role in terms of internal forces and stiffness of the truss.

Significant increase of the whole main truss displacements in the lateral direction has been calculated. The displacements of rafters of the main truss have increased to more than double value. In case of the secondary truss, where only the rafters, the collar beam and the tie beam resisted to the load actions, the calculated displacements reached approximately 3.8-times higher values. Deflection of the tie beam has increased up to 25% in case of the main truss, opposite to the tie beam of the secondary truss, which was not affected by the rafter behaviour.

Longitudinal truss

The stresses calculated in the upper chord, bottom chord and the upper X-bracing members of the longitudinal truss have increased approximately 2-times. Although the increase is relatively high, the stress values are low in terms of structural resistance considering the fact that the major part of stresses is induced by bending moments. The bracing crosses of the longitudinal truss have a function of longitudinal bracing of the roof. According the calculation results, the crosses have been also observed as the bracing member in the lateral direction of the truss, because these crosses are connected to the upper chord of the longitudinal truss and the central column. The bottom bracing crosses have been observed as the members with the lowest contribution to stiffness and force distribution. The increase of longitudinal truss displacements in vertical direction is approximately 20%.

Numerical model M2 vs. M0 – Absence of central angle braces

Main and secondary truss

The stresses in the rafters of the main truss have increased approximately 1.4-times, as a consequence of the decreased stiffness of the central column – to tie beam connection and the resulting tie beam deflection.

The increments of stresses occurred in the load combination, containing the wind actions.

Decreased stiffness of this connection caused in particular load combinations the decrement of stresses in the collar beam of the main truss, where lower influence of the central column on the collar beam was observed.

The stresses in the rafters, the collar beam, the tie beam members as well as the support angle braces of the secondary truss have not changed, what indicates an independent structural behaviour of the main and the secondary truss. It is important to note, that roofing layers causes the interactions between the main and the secondary trusses, but the loads in the numerical model are applied on particular independent rafters. On the other hand, the independent behaviour was partially expected because the gaps between the collar beams and the upper chord of the longitudinal truss were observed during the inspection of the roof trusses.

The stresses in the central column have slightly decreased. The support angle braces have been less influenced, when the increments of stresses up to 24% have been calculated at the main truss. The tie beam stresses of the main truss have been increased 1.5-times.

Significant increase of the whole main truss displacements in lateral direction has been calculated. The displacements of rafters of the main truss have increased approximately 1.4-times. Deflection of the tie beam of the main truss has increased by 21%, opposite to the tie beam of the secondary truss that was unaffected by the rafter behaviour.

Longitudinal truss

The stresses calculated in the upper chord and the upper X-bracing members of the longitudinal truss have increased approximately by 12–20%. The stresses calculated in the bottom chord of the longitudinal truss have increased approximately 1.6-times. Although the increase is relatively high, the stress values are low in terms of the structural resistance. The structural behaviour of the bracing crosses of the longitudinal truss played role in the lateral stiffness of the roof structure, as it was observed also in the numerical model M0, and M1. The effect of the bottom bracing crosses has been observed as a very low in terms of the stiffness and force distribution.

The maximum value of displacements calculated at the longitudinal truss in vertical direction has not been changed.

CONCLUSIONS

The research results point to a rational approach to the design of roof trusses used geometry and arithmetic. Unlike now, an approach to the planning of the structure service life was significantly different, looked through a number of other generations. For the design they used the perfect musical ratios and irrational proportions, which were considered to be perfect. The results showed that the roof structure safely satisfy (with reserve about 30%) the reliability conditions defined by current European standards for structural design, both in terms of the ultimate limit states and serviceability limit states. The reserve extends the service life in relation to the potential long-term damage to the life of roof truss constructions and enables their rehabilitation [Korenková 2013].

ACKNOWLEDGEMENT

This paper presents results of works supported by the Scientific Grant Agency of the Slovak Republic under the project No. 1/1296/12.

REFERENCES

Brunés, T. (1967). The Secrets of Ancient Geometry – and its uses. Rhodos International Science Publishers, Rhodos.
Korenková, R. (2013). The analysis of methods used in renovations of historical roof frames. Civil and environmental engineering, 9(2), 144–149.
Krušinský, P. et al. (2008). Historical trusses of the sacral buildings in the region of Turiec. M. Gibala, Kysucke Nove Mesto.
Palladio, A. (1797). Los quatro libros de arquitectura. Madrid.
Scia Engineer (2010). Software for structural analysis. http://nemetschek-scia.com/sk.
STN EN 338 Structural timber (2004). Strength classes. SÚTN, Bratislava.
STN EN 1990 Eurocode (2009). Basis of structural design. SÚTN, Bratislava.
STN EN 1991-1-1 Eurocode 1 (2007a). Actions on structures. Part 1-1: General actions. Densities, self-weight, imposed loads for buildings. SÚTN, Bratislava.
STN EN 1991-1-4 Eurocode 1 (2007b). Actions on structures. Part 1-4: General actions – Wind actions. SÚTN, Bratislava.
STN EN 1995-1-1 + A1 Eurocode 5 (2008). Design of timber structures. Part 1-1: General. Common rules and rules for buildings. SÚTN, Bratislava
Struhár, A. (1977). Geometric harmony of historical architecture in Slovakia. Pallas, Bratislava.
Suger from Saint-Denis (2006). Manuscripts about Saint-Denis (Original 1142). Oikoymeneh, Praha.
Vitruvius (2001). The ten books on architecture. Arista Baset, Praha.

ANALIZA HISTORYCZNEGO WIĄZANIA DACHOWEGO WE WSI BELÁ DULICE

Streszczenie. Powstanie wiązania dachowego kościoła rzymskokatolickiego pod wezwaniem Najświętszego Ciała Chrystusowego datuje się na rok 1409. Reprezentuje ono jedną z kilku dobrze zachowanych średniowiecznych konstrukcji w tym regionie. Spiczasty dach nad nawą ma charakterystyczną konstrukcję z jętką podpierającą krokwie z podłużnym usztywnionym wiązaniem dachowym. Analiza geometryczna głównego wiązania dachowego, jak również środkowego podłużnego wiązania dachowego opiera się na logicznych zależnościach oraz opisie procesu projektowania wiązania dachowego, wskazując na krytyczne relacje wynikające szczególnie z geometrii pitagorejskiej. W rezultacie opracowano przestrzenny model liczbowy struktury dachowej, żeby przeprowadzić analizę statystyczną tej struktury zgodnie ze współczesnymi standardami. W związku z tym, że podczas badań stwierdzono braki niektórych strukturalnych elementów konstrukcji dachu (klamry kątowe), w dalszych analizach położono nacisk na znaczenie wybranych elementów strukturalnych oraz ich rolę w konstrukcji wiązania dachowego jako takiego.

Słowa kluczowe: historyczne wiązanie dachowe, analiza geometryczna, niewymierna proporcja, analiza statystyczna, model liczbowy

Accepted for print – Zaakceptowano do druku: 17.03.2015

For citation: Gocál, J., Krušinský P., Capková, E., Kekeliak, M. (2015). Analysis of the historical truss in village Belá Dulice. Acta Sci. Pol., Formatio Circumiectus 14(1), 57–73.